Why derivatives on derivatives? The case of spread futures

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Abstract

Recently, calendar spread futures, futures contracts whose underlying asset is the difference of two futures contracts with different delivery dates, have been successfully introduced for a number of financial futures contracts traded on the Chicago Board of Trade. A spread futures contract is not an obvious financial innovation, as it is a derivative on a derivative security: a spread futures position can be replicated by taking positions in the two underlying futures contracts, both of which may already be quite liquid. This paper provides a motivation for this innovation, demonstrating how the introduction of spread futures can, by changing the relative trading patterns of hedgers and informed traders, affect equilibrium bid–ask spreads, improve hedger welfare, and potentially improve market-maker expected profits. These results are robust both to allowing serial correlation of asset price changes, and investor preference for skewness.
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1. Introduction

In January 2001, Alliance/CBOT/Eurex (a/c/e), a joint venture of the Chicago Board of Trade and Eurex futures exchanges, began trading four separate reduced tick spread futures contracts. The underlying asset for these futures is a calendar spread position across

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two otherwise identical CBOT futures contracts with adjacent delivery dates. Thus, these spread futures contracts are redundant securities in the sense that the contract essentially consists of a long position in one futures contract, and a short position in another futures contract, with the two legs of the spread differing only in their delivery dates. In fact, once entered into, the spread futures contract is treated by the exchange exactly the same as if the two legs of the spread had been entered into separately.

In another, narrower sense, reduced tick spread futures are perhaps not totally redundant: as implied by the name, these spread futures have a smaller allowed tick size than the associated primary futures contract, and thus can trade at prices unavailable by directly trading the associated long and short futures positions.[1] Additional institutional detail about available reduced tick spread futures contracts is provided in Section 3.

Since spread futures contracts are redundant in the sense that the underlying spread can be straightforwardly traded on other futures markets, it is not immediately clear why this particular financial innovation should be successful. Indeed, not only are spread futures contracts a derivative of other derivatives, the two underlying futures contracts composing the calendar spread, but the particular spread futures introduced to date are based on very liquid futures contracts. Why are these apparently redundant contracts observed? This paper addresses this question and suggests that this security changes the trading behavior and welfare of hedgers in such a way as to make this innovation potentially attractive to a futures exchange.

A model is provided to show how the structure of the transaction costs in the futures market, modeled in the form of bid–ask spreads, is changed by the introduction of calendar spread futures. Futures markets exist in the model to service hedging demand, an approach traceable to Working (1953). Also present are informed traders. Market-makers provide the market with liquidity, which is costly due to the adverse selection of facing informed traders. Market-makers are compensated through charging a bid–ask spread on trades. With competitive market-makers, bid–ask spreads in each contract are set to just cover the adverse selection cost faced by market-makers in that contract. However, if the cost of trading a calendar spread is lower in the spread futures than the primary futures market, then hedgers' trades will partially migrate to the spread futures market, leaving informed trading in the primary market. Furthermore, if the overall cost of implementing hedges falls, additional hedging interest may arise in the primary market. Introducing spread futures thus allows partial separation of hedging and informed trading. Trading in the spread futures market is concentrated in hedging, and therefore supports a lower bid–ask spread.

A similar result is obtained if bid–ask spreads are set by an exchange exercising pricing power in order to maximize aggregate market-maker profit. With these "monopolistic" bid–ask spreads, it is optimal to lower the bid–ask spread in the calendar spread futures, attracting hedgers, while raising the bid–ask spread in the primary market. This allows price discrimination between hedgers and informed traders. Informed traders face higher trading costs and reduce their activity, moderating the adverse selection problem faced

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[1] Of course, if reducing the tick size is the primary innovation associated with these spread futures, one could ask why not instead reduce the tick size on the underlying CBOT futures contracts. This paper shows that introducing spread futures leads to quite different results than reducing the tick size.

by the market-makers. The lower bid–ask spread in the spread futures market effectively subsidizes hedgers, keeping their overall trading cost relatively low in order to generate the hedging trades that market-makers find profitable.

To implement either such equilibrium, featuring a smaller bid–ask spread in the spread futures, requires a finer pricing structure, or reduced tick size, in the spread futures relative to the primary contract. Even without the presence of the spread futures, a trader could, in principle, negotiate both legs of the calendar spread simultaneously in the futures market. However, the possible prices at which the legs can be negotiated are constrained by the tick size. By allowing a smaller tick size, the presence of the spread futures market allows a finer set of possible prices, and therefore a smaller bid–ask spread (lower transaction cost) in trading the calendar spread.

The paper is organized as follows. Section 2 provides a literature review. Section 3 provides institutional detail about reduced tick spread futures. Section 4 describes the model, and includes results about bid–ask spreads, hedger volume, and hedger welfare, for both the cases of competitive and monopolistic bid–ask spreads. Three extensions are considered in Section 5, serial correlation of underlying asset price changes, hedger wealth preferences reflecting skewness, and whether the model should apply to the trading of calendar spreads on options. Section 6 concludes.

2. Literature review

Excellent overviews of the literature on financial innovation are provided in Allen and Gale (1994) and Duffie and Rahi (1995). Specific cases of innovations of futures contracts are discussed in Working (1953), Gray (1970), Sandor (1973), Silber (1981), and Johnston and McConnell (1989).

This paper reaches the conclusion that the introduction of spread futures, which appears to be a redundant security, can change trading patterns and hedger welfare. In the options literature, there is evidence that options may not be truly redundant. Conrad (1989) finds a significantly positive (two percent) abnormal stock return accompanying the introduction of stock option trading for listings from 1974 and 1980. Detemple and Jorion (1990) find similar abnormal stock returns for listings from 1973 to 1982, but no significant effect for listings from 1982 to 1986. Back (1993) models a market where an option can be synthesized via dynamic trading, thus appearing to be redundant, but the option's existence affects the information flow, making the underlying asset volatility stochastic. Longstaff (1995), for the case of S & P 100 index options, rejects the martingale restriction that the value of the underlying asset implied by the cross-section of options prices equals its actual market value, finding that the difference in value is related to market frictions. If markets are dynamically complete and options are redundant assets, then any option payoff can be replicated using the underlying asset and one additional option. Buraschi and Jackwerth (2001) perform this direct test, concluding that at-the-money S & P 500 index options and the underlying index do not span the pricing space; consequently, the options are not redundant securities. Bakshi et al. (2000) conclude that index options are not redundant assets, as the index level and associated call option prices often move in opposite directions.

There is also a literature on how the introduction of futures contracts may affect the underlying asset markets. A comprehensive summary of this literature is in Mayhew (2000). One related paper is Subrahmanyam (1991), which provides an information-based model for stock-index futures ("basket trading"), extending Kyle's (1984) model to allow simultaneous trading of individual stocks and baskets of stocks. With the introduction of basket trading, uninformed traders tend to trade the basket to protect themselves from the informed traders, who tend to trade individual stocks since their information is stock-specific. Thus, the model predicts liquidity will migrate from the markets for individual stocks to the basket.

3. Institutional detail

This section provides some specific detail about the currently traded reduced tick spread futures contracts. These futures contracts are traded exclusively on the a/c/e electronic trading platform, while their clearing is through the CBOT system. The underlying asset for these futures is a calendar spread position across two otherwise identical CBOT futures contracts with adjacent delivery dates: one long futures position and one short futures position, with the two legs of the spread differing only in their delivery dates.

As an example of a traded reduced tick spread future, the March 10-year US Treasury Note Futures Reduced Tick Spread futures contract has a trading unit that consists of "one March–June Ten-year US Treasury Note futures spread having a face value at maturity of $100,000 or multiple thereof."[2] Taking a long position in this spread futures contract immediately gives the trader a long position in the CBOT June 10-year US Treasury Note futures, and a short position in the CBOT March 10-year US Treasury Note futures, just as if these two positions were entered into directly, but separately, through the CBOT Treasury Note futures markets. Thus, for clearing purposes, a trade executed in a spread futures contract is recognized as being exactly the same as two trades in the futures contracts corresponding to the legs of the calendar spread.

Although the tick size of the CBOT 10-year US Treasury Note futures contract is one-half of 1/32 of a point, the tick size of the associated a/c/e Reduced Tick Spread futures contract is smaller, one-quarter of 1/32 of a point. Similarly, the tick size for the CBOT 5-year Treasury Note and (both 10-year and 5-year) Agency Note futures contracts is one-half of 1/32 of one point. The tick size for the CBOT Treasury Bond and Interest Rate Swap (both 10-year and 5-year) futures contracts is 1/32 of one point. The tick size of all the associated reduced tick spread futures is one-quarter of 1/32 of one point. Thus, the tick size for each spread futures contract is one-half or one-quarter the tick size of the associated futures contracts.

Because the spread futures contract price reflects the price differential between two futures contracts differing only in delivery date, one naturally expects the spread futures price to be much smaller than the price of either leg of the underlying spread. In light of this, implementing the reduced tick pricing may seem quite natural. Furthermore, the spread

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[2] Language taken from the description on the Chicago Board of Trade website http://www.cbot.com.

differential may even carry a negative price, depending upon the slope of the interest-rate term structure, for the case of interest-rate futures.[3]

To date, a/c/e has introduced seven reduced tick spread futures contracts. Three of these, associated with US Treasury Bond futures, 10-year Treasury Note futures, and 5-year Treasury Note futures, were introduced in January 2001, and have been well received in the marketplace. A 10-year Agency Note futures reduced tick spread futures contract was also introduced in January 2001. Although modestly successful at first,[4] the trading volume for both the underlying 10-year Agency Note futures and its associated reduced tick spread futures have migrated to the 10-year Swap futures and its associated reduced tick spread futures since the CBOT introduced the 10-year Swap futures contract in October 2001, followed by its reduced tick spread futures contract in May 2002. Two other reduced tick spread futures products, based on the 5-year Agency Note futures and 5-year Swap futures contracts, have also been introduced. However, neither of the two underlying futures contracts nor their associated reduced tick spread futures contracts has ever generated meaningful volume up to this time. Table 1 shows the monthly trading volume to date for the first five reduced tick spread futures contracts.

As is apparent from Table 1, there is seasonality in the volume for each of these contracts. The expiration month on all the underlying financial futures contracts is March, June, September, or December. The significant demand for rolling over futures contracts occurs in four weeks leading up to the contract expiration date. Closer examination of daily trading volume (not shown) reveals that, for each of these reduced tick spread futures, volume starts to visibly increase around the 15th to the 20th of the previous month, typically peaks on the 28th or the 29th, then visibly decreases around the 7th of the contract month. For example, for the March contract, the largest volume occurs from about February 15 through March 7.

4. The model

The model contains hedgers, informed traders, and market-makers in an overlapping generations-style marketplace. Risk neutral market-makers provide liquidity to futures markets by taking the opposite side of trades, as needed, and are compensated by capturing the difference between the bid and ask prices. Market-makers incur an adverse selection cost when trading against better informed traders.

Time is broken up into a series of trading periods (dates). There is a risky asset, whose underlying value changes between each period, with mean zero and variance σ^2. Value changes are independent over time in the basic model; serial correlation is allowed in one of the model extensions. A series of futures contracts is available, each of which is tradable for two consecutive periods, after which delivery takes place (although in the model, traders

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[3] a/c/e has developed a pricing convention responding to traders' presumed disinclination to work with negative prices. The convention is based on adding 100 basis points to the price differential for all reduced tick spread futures prices.

[4] The underlying 10-year Agency Note futures contract has also achieved only modest success since its introduction in 2000.

Table 1
Monthly volume of reduced tick spread futures, by contract

	US Treasury Bonds	Ten-year Treasuries	Five-year Treasuries	Ten-year Agencies	Ten-year Swaps
Jan. 2001	212	110	0	0	N/A
Feb. 2001	25,750	74,050	25,756	8117	N/A
Mar. 2001	11,410	21,196	9300	0	N/A
Apr. 2001	11,178	1696	0	0	N/A
May 2001	79,438	160,178	36,786	3000	N/A
Jun. 2001	24,260	59,588	41,272	0	N/A
Jul. 2001	774	1298	0	0	N/A
Aug. 2001	147,530	221,256	84,920	2452	N/A
Sep. 2001	33,940	156,135	26,008	2276	N/A
Oct. 2001	8920	1322	4874	304	N/A
Nov. 2001	101,800	189,346	79,612	3558	N/A
Dec. 2001	24,356	58,204	54,320	992	N/A
Jan. 2002	1838	2020	900	0	N/A
Feb. 2002	146,946	233,906	177,262	2388	N/A
Mar. 2002	133,184	163,432	54,970	0	N/A
Apr. 2002	300	4006	150	0	N/A
May 2002	230,174	380,750	234,304	286	7702
Jun. 2002	96,836	158,966	102,608	0	0
Jul 2002	8740	32,578	676	0	0
Aug. 2002	219,074	423,324	177,700	0	2502
Sep. 2002	116,844	191,996	52,898	166	8314
Oct. 2002	11,562	11,136	752	0	0
Nov. 2002	272,150	588,564	416,586	0	23,524
Dec. 2002	125,444	260,522	125,730	0	0
Jan. 2003	15,262	66,586	11,624	0	0
Feb. 2003	355,406	778,548	473,828	0	20,480

Note: Reduced tick spread futures were introduced in January 2001 for 30-year Treasuries, 10-year Treasuries, 5-year Treasuries, and 10-year Agencies, and introduced in May 2002 for 10-year Swaps.

clear their positions before delivery occurs). Futures contracts overlap, so that in each trading period, two futures contracts with different delivery dates are extant: traders can take positions in the "new" contract (delivery immediately after next period), and close out positions in the "old" contract (delivery immediately after this period).

Each period, there is a mass H of new hedgers. Each hedger is equally likely to be endowed with $+E$ or $-E$ units of the risky asset. Each hedger's lifetime is either one or two periods long; q is the probability of a two period lifetime. A hedger can trade in the futures market(s) for the risky asset during his life. Thus, a hedger with a one period lifetime born at date T can initiate a futures position at date T, and close the position at date $T+1$, while a hedger with a two period lifetime can initiate a futures position at date T, adjust it at date $T+1$, and close it at date $T+2$. Hedgers do not know their lifetime at birth, but find it out after one period passes. Hedgers have mean-variance preferences with risk-aversion Γ over their final wealth.

Each period, there is a mass I of new informed traders. Each informed trader has private information about the next risky asset value change. Conditional on her information, an informed trader either appraises the next risky asset value change as having mean $+\theta > 0$

or mean $-\theta$, ex ante equally likely, and variance $k\sigma^2$. Therefore, an informed trader's gain from making a unit trade, of appropriate direction, in the risky asset has mean θ and variance $k\sigma^2$, less the bid–ask spread.[5] The parameters θ and k thus measure the mean and variance of the quality of informed traders' information. Informed traders have one period lifetimes and mean-variance preferences with risk-aversion Γ over their final wealth.

Two scenarios are considered: a scenario with the two already described futures contracts, new and old, trading each period ("without spread futures"), and an alternative scenario with a spread futures contract available as well ("with spread futures"). The spread futures contract is based on the calendar spread between the new and old contracts, and is identical to a long position in the new contract combined with a short position in the old contract. When available to trade, the spread futures contract will be the natural instrument for hedgers who discover their lifespan is two periods, leading to a desire to roll over their short-lived position into a longer-lived position. Utilizing spread futures allows these hedgers to convert their position in the old contract, which delivers before the conclusion of their hedging needs, into a position in the new contract, which delivers after the conclusion of their hedging needs. Of course, if the spread futures contract is unavailable, the hedgers can achieve a similar result directly by simultaneously trading equal and opposite positions in the new and old futures contracts.

When trading, traders incur, and market-makers receive, a bid–ask spread, the frictional cost associated with a round trip trade for a hedger or informed trader. The endogenously determined bid–ask spread in the futures for the risky asset is denoted by $S \geqslant 0$. When spread futures are available, the bid–ask spread in the spread futures is denoted by $D \geqslant 0$. Since the calendar spread can always be directly created in the futures markets, $D \leqslant S$.

Suppose spread futures are available to trade. A hedger with endowment $-E$ optimally takes an initial futures position $x[S, D]$, adjusted the next period to a position $x'[S, D, x]$ if the hedger turns out to be long-lived. These positions satisfy

$$\underset{x,x'}{\text{Max}} \ (1-q)\big[-S|x| - \Gamma\sigma^2(x-E)^2/2\big]$$
$$+ q\big[-S \cdot \max(|x|, |x'|) - D \cdot \min(|x|, |x'|) - \Gamma\sigma^2\big((x-E)^2 + (x'-E)^2\big)/2\big]. \tag{1}$$

At the optimum, these are identical long positions $x' = x \geqslant 0$. Symmetrically, for a hedger with endowment $+E$, the optimal initial and adjusted futures positions are the short positions of the same magnitude, $-x$. Therefore, all H hedgers take an initial futures position with magnitude $x[S, D]$. Of these hedgers, qH will be long-lived and roll over their position next period using spread futures, while $(1-q)H$ will be short-lived and simply close their position next period.

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[5] One possible underlying structure is that the underlying risky asset value change is equally likely to be $+\sigma$ or $-\sigma$ each period, and the informed trader, after observing her signal, assigns probability $p > 1/2$ to the correct direction of price movement. Then $\theta = p\sigma - (1-p)\sigma = (2p-1)\sigma$, and $k = 4p(1-p)$. Another possible underlying structure is that the asset value change each period takes the form $\Theta + \Phi$, with Θ equally likely to be $+\theta$ or $-\theta$, and Φ independent of Θ. If the informed perfectly observes Θ, then $k = 1 - (\theta/\sigma)^2$.

An informed trader with conditional mean $+\theta$ optimally takes futures position $y[S]$ satisfying

$$\underset{y}{\text{Max}}\ (\theta - S)y - \Gamma k\sigma^2 y^2/2 \tag{2}$$

if $S \leqslant \theta$. If the bid–ask spread is so large that $S > \theta$, then informed traders do not participate in the market, $y[S] = 0$. Similarly, an informed trader with conditional mean $-\theta$ optimally takes futures position $-y[S]$. Thus, $y[S]$ is the magnitude of the futures position for all I informed traders.

Alternatively, suppose spread futures contracts are not available for trade. The same calendar spread trade can be made directly using the primary futures contracts, incurring a bid–ask spread S. Trades of both hedgers and informed can be inferred by setting $D = S$ in the previous analysis. All H hedgers take initial futures positions with magnitude $x[S, S]$. All hedgers will close this position next period, but qH of the hedgers will also open the same size position in the subsequent delivery futures contract. As before, all I informed traders take a position of magnitude $y[S]$.

4.1. Competitive bid–ask spreads

This section considers the case of the bid–ask spread(s) for futures contracts being set competitively, so that market-makers break even in expectation. Without spread futures, the break-even condition in the primary futures market is

$$H(1 + q)S \cdot x[S, S] + I(S - \theta) \cdot y[S] = 0. \tag{3}$$

Here, H hedgers generate trade volume $H(1 + q) \cdot x[S, S]$, and I informed traders generate volume $I \cdot y[S]$. Market-makers receive bid–ask spread S for each contract traded, but expect to lose θ per contract traded with informed traders due to information asymmetry. In the absence of spread futures, denote the competitive bid–ask spread in the primary futures market, satisfying (3), by S_{NSF}.

With spread futures available, the break-even conditions in the primary and spread futures markets are

$$HS \cdot x[S, D] + I(S - \theta) \cdot y[S] = 0, \qquad HqD \cdot x[S, D] = 0. \tag{4}$$

The H hedgers and I informed traders generate volume $H \cdot x[S, D] + I \cdot y[S]$ in the primary futures, and $qH \cdot x[S, D]$ in the spread futures. Market-makers receive bid–ask spread S per primary and D per spread futures contract traded, but expect to lose θ per primary futures contract traded with informed traders due to information asymmetry. In the presence of spread futures, denote the competitive bid–ask spreads in the primary and spread futures markets, satisfying (4), by S_{SF} and D_{SF}, respectively.

To avoid the possibility that the adverse selection problem is so severe that it shuts down the futures market, it is assumed that $\theta \leqslant \Gamma\sigma^2 E$. By implying the existence of a primary market bid–ask spread large enough to eliminate all informed trade, but not all hedging activity,[6] this guarantees the existence of bid–ask spreads sustaining futures trade. This

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[6] Specifically, any spread S satisfying $\theta \leqslant S < \Gamma\sigma^2 E$ suffices.

is consistent with the focus of the paper, examining the introduction of calendar spread futures to an already existing futures market.

As long as the available spread futures market offers a lower transaction cost than direct trading, $D < S$, hedgers desiring a calendar spread utilize spread futures rather than constructing the spread by simultaneously trading the new and old futures contracts. Hedgers rolling over old positions migrate to the spread futures market while hedgers initiating new positions as well as informed traders remain in the primary futures market. However, hedgers recognize that the potential cost of rolling over their position later is lower and therefore take larger initial futures positions.

Since only hedgers migrate to the spread futures market, the bid–ask spread in that market falls. In the model, since adverse selection generates the only trading friction, the bid–ask spread falls all the way to zero. Remaining in the primary market are informed traders and hedgers taking initial positions, although the latter increase their position size. Since hedgers generate two opposing demand effects in the primary market, the bid–ask spread there can either increase or decrease. Specifically, if the adverse selection problem is sufficiently strong, so that a large bid–ask spread is required in the equilibrium without spread futures, then introducing spread futures lowers the trading cost for hedgers, and increases market-maker revenue from hedgers; lowering the bid–ask spread in the primary market increases market-makers losses from informed, and brings the market-maker to the break-even profit level. If the adverse selection problem is weak, with a small bid–ask spread in equilibrium without spread futures, introducing spread futures decreases market-maker revenue from hedgers; raising the bid–ask spread in the primary market decreases market-maker losses from informed, bringing the market-makers to break even. The adverse selection problem is weak when the informed signal has low mean θ and high variance k, and the number of informed traders per hedger I/H is small.

The separation of hedgers from informed traders through spread futures lessens the adverse selection problem, and the benefits pass to hedgers with competitive market-making. Hedgers care about only their overall (initial plus adjustment) trading cost $S + qD$, which declines. Total (primary and spread) hedging volume $H(1 + q)x$ increases, and hedgers are better off. This intuition is formalized in Proposition 1.

Proposition 1. *The competitive bid–ask spread of the primary futures market may be increased or decreased by the introduction of calendar spread futures. The bid–ask spread is increased, $S_{\text{SF}} > S_{\text{NSF}}$, exactly when $\theta < \Gamma \sigma^2 E \cdot [1 + \sqrt{(kH/I)}] \cdot (1+q)/(2+q)$. The competitive bid–ask spread of the calendar spread futures is $D_{\text{SF}} = 0$. Total hedging volume is higher in the presence of spread futures, as $x[S_{\text{SF}}, D_{\text{SF}}] \geqslant x[S_{\text{NSF}}, S_{\text{NSF}}]$. Hedgers are made better off by the introduction of calendar spread futures.*

In order to implement the new competitive bid–ask spreads with the introduction of calendar spread futures, it may be necessary to reduce the minimum tick size. In particular, if the minimum tick size in the primary market was set near the competitive bid–ask spread without spread futures, then the minimum tick size in the spread futures will need to be set below that of the primary market. Thus, it is natural to expect that the spread futures will be introduced with the reduced tick feature.

4.2. Monopolistic bid–ask spreads

The section considers the case where the exchange is able to exercise monopoly power in setting the levels of the bid–ask spread(s) for futures contracts. Furthermore, the bid–ask spreads are set to maximize the aggregate profit of the market-makers on the exchange, who may, for example, be owners of the exchange. In the case without spread futures, the optimization problem is

$$\underset{S}{\text{Max}} \; H(1+q)S \cdot x[S, S] + I(S - \theta) \cdot y[S]. \tag{5}$$

In the case with spread futures, the optimization problem is

$$\underset{D \leqslant S}{\text{Max}} \; H(S + qD) \cdot x[S, D] + I(S - \theta) \cdot y[S]. \tag{6}$$

The $D \leqslant S$ constraint is necessary because if spread futures are not cheaper to trade, then traders can implement calendar spreads by trading directly in the primary market.

Similar to the competitive case, when a spread futures contract with lower transaction cost is available, traders sort themselves by market. Hedgers rolling over old positions use spread futures, while hedgers initiating new positions as well as informed traders use the primary futures market. The lower potential cost of rolling over positions entices hedgers to take larger initial futures positions.

The presence of spread futures allows market-makers to (third degree) price discriminate between hedgers and informed traders. Hedgers incur a cost S in taking their initial futures position and an additional cost $D < S$ if they roll the position over. Thus, the average trading cost for hedgers, including trading in both futures markets, is less than S, while the average trading cost for informed traders is S since they cannot effectively utilize spread futures. By price discriminating, market-makers can make trading more attractive for the desired hedgers and less attractive for the undesired informed traders. Price discrimination will be most effective by emphasizing a large difference in average trading cost between hedgers and informed, increasing S and decreasing D. Optimally, the bid–ask spread D for calendar spread futures is set at its lower limit of zero, and the bid–ask spread S for the primary futures is set above its level in the absence of spread futures. With the introduction of spread futures, the total trading cost for hedgers decreases, so the size of their initial positions as well as the total hedging volume increases. This is formalized in Proposition 2. An asterisk is used to denote monopolistic bid–ask spreads.

Proposition 2. *The monopolistic bid–ask spread of the primary futures market is increased by the introduction of calendar spread futures, $S_{SF}^* \geqslant S_{NSF}^*$. The monopolistic bid–ask spread of the calendar spread futures is $D_{SF}^* = 0$. Total hedging volume is higher in the presence of spread futures, as $x[S_{SF}^*, D_{SF}^*] \geqslant x[S_{NSF}^*, S_{NSF}^*]$. Hedgers are made better off by the introduction of calendar spread futures.*

The monopolistic bid–ask spread in the primary futures market is determined by the tension between two opposing forces, the simultaneous desires to choose the bid–ask spread to maximize revenue from hedgers, and to minimize informed trade and its associated adverse selection problem. This tension exists whether or not calendar spread futures are

present. The key variable is the average trading cost for hedgers; this determines the total hedging volume across the futures markets as well as the revenue market-makers receive from hedgers. Without spread futures, the average trading cost for hedgers is simply the primary futures bid–ask spread; with spread futures, it also includes the lower spread futures bid–ask spread.

To maximize revenue from hedgers, the average hedger trading cost should be set proportional to the hedger endowment size E (which is essentially the intercept of a linear demand curve in the presence of a monopolist), whether or not spread futures trade. To minimize adverse selection, the primary futures bid–ask spread should be set equal to θ, the informed trader informational advantage. This translates to an average hedger trading cost of θ without spread futures, and less than θ with spread futures, as hedgers then also use lower-cost spread futures. When E is large relative to θ, so the bid–ask spread is greater than the informational advantage of traders, the adverse selection problem is eliminated; average hedger trading costs and hedging volumes are identical with or without spread futures. When E is small relative to θ, the average hedger trading cost is set at a weighted average of the "maximize revenue" and "minimize adverse selection" levels; average hedger trading cost is lower and hedging volume is higher with spread futures.

As with competitive bid–ask spreads, even if the minimum tick size was near the optimal bid–ask spread before the introduction of spread futures, implementing optimal bid–ask spreads with calendar spread futures requires the spread futures to offer a reduced tick, relative to the primary futures market. Furthermore, since the optimal bid–ask spread in the primary market increases, only the spread futures, and not the primary market, need reduce its tick size.

Because the monopolist has an additional parameter to optimize over, the aggregate market-maker profit with spread futures, π^*_{SF}, is always at least as large as the profit without spread futures, π^*_{NSF}. The gain of the market-makers, $\Delta\pi^* = \pi^*_{\text{SF}} - \pi^*_{\text{NSF}}$ naturally depends upon the underlying parameters. The comparative statics with respect to the various parameters are given in Proposition 3.

Proposition 3. *Under monopolistic bid–ask spreads, market-makers gain by the introduction of calendar spread futures. The magnitude of the aggregate market-maker gain $\Delta\pi^* = \pi^*_{\text{SF}} - \pi^*_{\text{NSF}}$ is increasing in the number of hedgers H, the number of informed traders I, and the mean of information quality θ. The gain is decreasing in the size of hedging needs E, variance of informed quality k, trader risk aversion Γ, and risky asset volatility σ.*

From the market-makers' viewpoint, introducing spread futures trading allows price discrimination against informed traders in terms of the bid–ask spread faced. Thus, the market-makers' gain depends upon the magnitude of the adverse selection problem generated by the informed traders. This is increasing in the number of informed traders I, the expected informational advantage θ (signal mean) of such a trader, and is decreasing in the noisiness of the information as measured by k (signal variance), informed trader risk aversion Γ and underlying asset volatility σ. The primary source of market-maker profit is the bid–ask spread received from trading against hedgers; thus the aggregate market-maker gain is increasing in the number of hedgers H.

Comparative statics with respect to E, the endowment size per hedger, is more compli- cated. As previously described, when E is large relative to θ, the average hedger trading cost is identical with and without calendar spread futures; the aggregate market-makers gain from the introduction of spread futures is zero. When E is small relative to θ, the average hedger trading cost is lower with spread futures, leading to a more significant dif- ference in aggregate market-maker profits for the cases with and without spread futures. Thus, as E becomes smaller relative to θ, the aggregate gain of the market-makers from the introduction of spread futures becomes larger.

Of all these parameters, perhaps the two most likely in practice to determine whether a particular futures contract offers a relatively large magnitude gain for market-makers, and therefore an attractive opportunity for an exchange to innovate, are large values for H or I, corresponding to a large number of hedgers or informed traders, respectively. Note also that a large value of H generally translates into a large volume of hedging-based trading, while a large value of I generally translates into a large volume of information- based trading. Therefore, recognizing that designing and implementing a new security is a potentially expensive process, the introduction of a spread futures contract is most likely to make economic sense when the primary futures market already exhibits high volume. This is consistent with the observation that the spread futures introduced by the a/c/e to date are based on some of the most popular futures contracts, as measured by volume, traded on the Chicago Board of Trade.

5. Extensions

In this section, three extensions of the model are considered. In the first, serial corre- lation of price changes in the underlying asset are allowed. In the second, hedger wealth preferences are allowed to depend not only upon mean and variance, but skewness as well. Third, it is discussed whether this model of calendar spread futures could equally well be applied to the trading of calendar spreads on options.

5.1. Serial correlation of price changes

The model is now extended by allowing serial correlation of changes in the risky asset price. Specifically, changes ΔP_{t+1} in the risky asset price from t to $t+1$ follow a first- order autoregressive process $\Delta P_{t+1} = \rho \cdot \Delta P_t + \varepsilon_{t+1}$. The first-order autocorrelation of price changes is $\text{Corr}(\Delta P_t, \Delta P_{t+1}) = \rho$. It is assumed that the futures price equals the expectation of the risky asset price at contract delivery. It follows that the futures price change from time t to $t+1$ equals ε_{t+1}.

A hedger with endowment $-E$ of the risky asset optimally takes an initial futures posi- tion $x[S, D]$, adjusted the next period to a position $x'[S, D, x]$ if the hedger turns out to be long-lived. These positions satisfy

$$\operatorname*{Max}_{x,x'} (1-q)\big[-S|x| - \Gamma \sigma^2 (x-E)^2/2\big]$$
$$+ q\big[-S \cdot \max(|x|,|x'|) - D \cdot \min(|x|,|x'|) - \Gamma \sigma^2 \big((x-(1+\rho)E)^2$$
$$+ (x'-E)^2\big)/2\big]. \tag{7}$$

Symmetrically, a hedger with endowment $+E$ takes the opposite futures positions, $-x$ and $-x'$. For positive correlation $\rho > 0$, the futures positions satisfy $|x| \geqslant |x'|$; if a hedger turns out to be long-lived, he may only roll over part of his hedge. For negative correlation $\rho < 0$, the positions satisfy $|x| \leqslant |x'|$; if a hedger turns out to be long-lived, he may actually expand his hedge. The intuition is clearest for $q = 1$, $\rho = -1$, and zero transaction costs; a hedger, knowing himself long-lived, and that any price movement in the first period will be completely retracted in the second, effectively faces no first period price risk, so holds no hedge in the first period ($x = 0$) although he will hedge in the next period ($x' > 0$). Informed trader behavior is the same as the basic model.

Because hedgers may change their hedge size, the break-even conditions for competitive bid–ask spreads, (3) and (4), need adjustment. Without spread futures, competition in the primary futures market implies

$$HS(x + qx') + I(S - \theta)y = 0, \tag{8}$$

while with spread futures, competition in the primary and spread futures markets imply

$$HS \cdot \max(x, (1 - q)x + qx') + I(S - \theta)y = 0,$$
$$HqD \cdot \min(x, x') = 0. \tag{9}$$

With serial correlation, the appropriate constraint on the magnitude of the adverse selection problem is $\theta \leqslant \Gamma\sigma^2 E(1 + q + \rho q)/(1 + q)$, rather than the $\theta \leqslant \Gamma\sigma^2 E$ assumed in the basic model. As before, this is determined by the bid–ask spread where hedging demand drops to zero.

The basic intuition is the same as the basic model with competitive market-makers. Introducing spread futures allows migration of hedger spread trading, leaving informed and initial hedge trading in the primary market. The combination of larger initial hedges but less spreading in the primary market may lead to a higher or lower bid–ask spread there. The lower overall cost of hedging increases hedging volume and makes hedgers better off.

Proposition 4. *Allowing serial correlation, the competitive bid–ask spread of the primary futures market may be increased or decreased by the introduction of calendar spread futures. The bid–ask spread is increased, $S_{SF} > S_{NSF}$, exactly for $\theta < \Gamma\sigma^2 E \cdot [1 + \sqrt{(kH/I)}] \cdot (1 + q + \rho q)/(2 + q)$ and $\rho \geqslant -1/2$, or $\theta < \Gamma\sigma^2 E \cdot [1 + \sqrt{((kH/I)(1 - (1 + 2\rho)q^2/(1 + \rho q)))}] \cdot (1 + \rho q)/(2 - q)$ and $\rho < -1/2$. The competitive bid–ask spread of calendar spread futures is $D_{SF} = 0$. Total hedging volume is higher and hedgers are better off with the introduction of spread futures.*

In the case of monopolistically set bid–ask spreads, some results differ from the basic model because of long-lived hedgers adjusting their hedge, increasing or decreasing the size, depending upon whether serial correlation is negative or positive. Bid–ask spreads then allow market-makers to use price discrimination against hedgers. If hedgers simply roll over their hedge, only the total transaction cost $(S + qD)$ matters to them. However, if their hedge size varies over time, the relative level of the transaction costs (primary S and calendar spread D) matters as well. Monopolistic setting of the bid–ask spreads may then allow wealth to be extracted from hedgers.

The optimization problem in the case without spread futures is

$$\underset{S}{\text{Max}} \ HS(x + qx') + I(S - \theta)y, \tag{10}$$

and in the case with spread futures is

$$\underset{D \leqslant S}{\text{Max}} \ H\big((1 - q)Sx + qS \cdot \max(x, x') + qD \cdot \min(x, x')\big) + I(S - \theta)y. \tag{11}$$

Intuition from the basic model still holds. When the spread futures offer a lower bid–ask spread, hedgers partially migrate to spread futures. By raising the bid–ask spread in the primary futures and lowering it in the spread futures, market-makers can make trading more attractive for hedgers and less attractive for undesired informed traders.

However, market-makers can also (second degree) price discriminate against hedgers. Hedgers face one price to set up a hedge and another to roll it over. In the basic model, only the total trading cost matters since hedgers keep the same size hedge over time. With serial correlation, hedgers may change their hedge size over time. Therefore, market-makers can use the relative prices of setting up a hedge (S) and rolling it over (D) to extract wealth from hedgers, engaging in second degree price discrimination. For parameters where the informed trader adverse selection problem is relatively unimportant, market-makers will focus on price discriminating against hedgers, and hedgers will be worse off with spread futures. Furthermore, since bid–ask spreads are set to emphasize second degree price discrimination against hedgers, rather than third degree price discrimination between hedgers and informed, the bid–ask spread for calendar spread futures need not be set as small as possible. These results are summarized in Proposition 5.

Proposition 5. *Allowing serial correlation, the monopolistic bid–ask spread of the primary futures market is increased by the introduction of calendar spread futures, $S^*_{\text{SF}} \geqslant S^*_{\text{NSF}}$. The monopolistic bid–ask spread of the calendar spread futures D^*_{SF} may be positive or zero. Total hedging volume is higher in the presence of spread futures. Hedgers may be made better off or worse off by the introduction of calendar spread futures.*

5.2. Skewness preference

In this section, instead of allowing serial correlation in risky asset price changes, hedger wealth preferences are allowed to depend not only upon mean and variance, but skewness as well.

With spread futures available, a hedger with endowment $-E$ of the risky asset and skewness preference coefficient $+\gamma$ optimally takes an initial futures position x, adjusted the next period to x' if the hedger turns out to be long-lived, satisfying

$$\underset{x,x'}{\text{Max}} \ (1 - q)\big[-S|x| - \Gamma\sigma^2(x - E)^2/2 - \gamma Q(x - E)^3/3\big]$$
$$+ q\big[-S \cdot \max(|x|, |x'|) - D \cdot \min(|x|, |x'|) - \Gamma\sigma^2\big((x - E)^2 + (x' - E)^2\big)/2$$
$$- \gamma Q\big((x - E)^3 + (x' - E)^3\big)/3\big], \tag{12}$$

where Q is the skewness of risky asset price changes. The optimal hedge positions are long and identical; write their magnitude as x_1. Similarly, a hedger with endowment $+E$

and coefficient $+\gamma$ takes short and identical optimal positions; write $-x_2$. A hedger with endowment $-E$ and coefficient $-\gamma$ optimally takes long positions x_2, and a hedger with endowment $+E$ and coefficient $-\gamma$ optimally takes short positions $-x_1$. Assuming an equal number of hedgers of each of the four types, hedgers are equally likely to be long and short; write the average position as $x[S, D]$.[7]

With skewness preference, the appropriate constraint on the magnitude of the adverse selection problem is $\theta \leqslant \Gamma \sigma^2 E - \gamma Q E^2$, rather than the $\theta \leqslant \Gamma \sigma^2 E$ assumed in the basic model. As before, this is determined by the bid–ask spread where hedging demand disappears.

In the case where bid–ask spread(s) are set competitively, the results and intuition are similar to the basic model. Introducing spread futures allows migration of hedger spread trading, leaving informed and initial hedge trading in the primary market. When the adverse selection problem from informed traders is strong (high mean θ and low variance k of informed signal, and a large number of informed per hedger I/H), the bid–ask spread in the primary market falls. Because long-lived hedgers roll over their entire hedge, their welfare depends upon the total hedger trading cost $S + qD$, which is $(1 + q)S_{\text{NSF}}$ without spread futures, and S_{SF} with spread futures. Their lower total cost implies higher hedger volume and hedger welfare under spread futures.

Proposition 6. *Allowing skewness preferences of hedgers, the competitive bid–ask spread of the primary futures market may be increased or decreased by the introduction of calendar spread futures. The bid–ask spread is increased, $S_{\text{SF}} > S_{\text{NSF}}$, when the adverse selection of informed is weak (θ, I/H, or $1/k$ sufficiently small). The bid–ask spread is decreased, $S_{\text{SF}} < S_{\text{NSF}}$, when the adverse selection of informed is strong (θ, I/H, and $1/k$ sufficiently large). The competitive bid–ask spread of calendar spread futures is $D_{\text{SF}} = 0$. Total hedging volume is higher and hedgers are better off with the introduction of calendar spread futures.*

In the case where bid–ask spread(s) are set monopolistically, the results and intuition are also similar to those of the basic model. Since hedgers roll over their entire hedge, market-makers are unable to second degree price discriminate against hedgers. The bid–ask spread in the primary futures market is (weakly) increased when calendar spread futures are introduced, but the overall trading cost faced by hedgers decreases, improving hedger welfare. The bid–ask spread for calendar spread futures is set as low as possible.

Proposition 7. *Allowing skewness preferences of hedgers, the monopolistic bid–ask spread of the primary futures market is increased by the introduction of calendar spread futures, $S_{\text{SF}}^* \geqslant S_{\text{NSF}}^*$. The monopolistic bid–ask spread of the calendar spread futures is $D_{\text{SF}}^* = 0$. Total hedging volume is higher, and hedgers are better off with the introduction of calendar spread futures.*

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[7] The model also works assuming all hedgers have the same skewness preference coefficient γ (or $-\gamma$), although equality of long and short position magnitudes is then lost. Thus, skewness preference need not imply either desire for or against skewness, just that skewness enters into hedger preferences.

Thus, the basic model is robust to allowing the preferences of hedgers to be extended from mean and variance of wealth to include skewness. The results are essentially the same as in the basic model, both for competitive and monopolistic spread setting.

5.3. Calendar spreads on options

It is worthwhile considering whether the model for calendar spread futures is likely to also apply to calendar spread options, that is, whether an innovation of explicitly traded calendar spread option contracts would be likely to be successful. There are three reasons to expect such an innovation to be distinctly less popular than for futures.

This paper models a naturally arising hedging demand for calendar spread futures, as hedgers who find their hedging horizon is greater than expected will extend the life of their original futures hedge by trading a calendar spread futures position. However, there is not a natural informed trader demand for calendar spread futures, since informed traders are more likely to want to take outright long or short positions.

Suppose instead there are hedgers with uncertain time horizon who prefer using options, rather than futures, to hedge. If they subsequently find their hedging horizon is greater than expected, will they actually wish to trade a calendar spread option position, simultaneously taking a distant expiry option position and unwinding a near expiry option position, and will such trades generate significant volume in aggregate? As mentioned, there are three reasons to expect such spread hedging demand to be distinctly less for options than for futures.

First, this hedging demand for calendar spread options will be diffused over a range of option strike prices, each corresponding to a different calendar spread option, depending upon the strike price of the original hedge. These strikes will depend upon historical price movements of the underlying asset and may be far from the current underlying price. This decreases hedging demand for any particular (strike price) calendar spread option. Second, for some option strike prices, the hedger may not wish to update his hedge with additional option positions. For example, to update a hedge whose strike price is now deeply in-the-money, the hedger may prefer using futures to options. For a hedge whose strike price is now deeply out-of-the-money, the hedger may be tempted to go unhedged for the remaining time. Therefore, hedging demand may be decreased even further for calendar spreads with extreme strike prices. Third, futures positions that are not unwound eventually face delivery (this argument assumes non-cash delivery), while options expiring out-of-the-money will just be left unexercised. Therefore, when hedging with options rather than futures, unwinding the original hedge when extending the hedge life becomes less critical. Hedgers who find their hedging horizon greater than expected may be more willing to simply buy the farther-dated option position than to employ a calendar spread trade. Again, this reduces the natural hedging demand for calendar spread option trading. With a less robust natural hedging demand, the appeal of innovating explicit trade in calendar spread options can be expected to be less than for calendar spread futures.

6. Conclusion

Calendar spread futures are a derivative on a derivative, equivalent to simultaneous long and short positions in two otherwise identical futures contracts with adjacent delivery dates. This paper examines why such an apparently redundant innovation is observed. Calendar spread trading is natural for hedgers who wish to extend the life of their original hedge, while informed traders are more likely to prefer trading directly in the primary futures market. Therefore, introducing calendar spread futures allows partial separation of hedger and informed trading. Spread futures trading disproportionately consists of hedgers, lessening the market-makers' adverse selection problem in that market.

The implications are derived both for the case where bid–ask spreads are set competitively and the case where an exchange exercises monopoly power in setting bid–ask spreads to benefit market-makers. Both cases result in a lower bid–ask spread for calendar spread futures relative to the primary futures market, overall lower transaction costs for hedgers, increased hedging volume and increased hedger welfare. In the monopolistic case, the bid–ask spread in the primary futures market increases, while in the competitive case, the bid–ask spread may increase or decrease. These results are relatively robust to changes in the model specification, including allowing serial correlation of price changes in the underlying asset, and allowing hedger wealth preferences to depend on skewness in addition to mean and variance. One notable change is that with serial correlation, hedgers optimally change their hedge size over time. This exposes them to the possibility of second degree price discrimination and reduced welfare in the monopolistic case.

Three types of empirical predictions arise from the model, relating to contract existence, trading volume, and the bid–ask spread. Futures markets which already exhibit high trading volume are predicted to be the most likely candidates for the introduction of calendar spread trading. In contrast, it is much less likely that formal contracts for trading calendar spread option contracts will arise. Futures in which hedgers wish to take longer-term hedges, and in which the adverse selection problem from informed investors is stronger, are also more likely candidates for spread trading.

Introducing calendar spread trading is predicted to increase the hedging volume associated with those futures. Although the bid–ask spread can either increase or decrease in the primary futures market, the model predicts that the adverse selection problem from informed trading should be smaller in the spread futures market, leading to a lower bid–ask spread than in the primary futures market. If the bid–ask spread in the primary market is near the tick size, then implementing the smaller bid–ask spread of spread futures requires the reduced tick feature observed in currently traded calendar spread futures contracts.

Appendix A. Proofs

Proof of Proposition 1. It is convenient in the proofs to define $e = \Gamma\sigma^2 E$ and $J = I/kH$. Optimizing (1), the adjusted futures position of a hedger with endowment $-E$ is

$$x'[S, D, x] = \begin{cases} E - S/\Gamma\sigma^2 & \text{if } x \leqslant E - S/\Gamma\sigma^2, \\ E - D/\Gamma\sigma^2 & \text{if } x \geqslant E - D/\Gamma\sigma^2 \geqslant 0, \\ 0 & \text{if } x \geqslant 0 \geqslant E - D/\Gamma\sigma^2, \\ x & \text{otherwise.} \end{cases}$$

The optimal initial position is

$$x[S, D] = \begin{cases} E - (S + qD)/\Gamma\sigma^2(1+q) & \text{if } S + qD \leqslant (1+q)\Gamma\sigma^2 E, \\ 0 & \text{otherwise,} \end{cases}$$

and only the fourth case for x' is optimal, $x' = x$. Similarly, the initial and adjusted futures positions of a hedger with endowment $+E$ are $-x$ and $-x$, so all hedgers take positions of magnitude x. In the case of no available spread futures,

$$x[S, S] = \begin{cases} E - S/\Gamma\sigma^2 & \text{if } S \leqslant \Gamma\sigma^2 E, \\ 0 & \text{otherwise.} \end{cases}$$

Optimizing (2), the magnitude of the initial futures position for an informed trader is

$$y[S] = \begin{cases} (\theta - S)/\Gamma\sigma^2 k & \text{if } S \leqslant \theta, \\ 0 & \text{otherwise.} \end{cases}$$

For a competitive futures exchange without spread futures, aggregate market-maker profit $H(1+q)S \cdot x[S, S] + I(S - \theta) \cdot y[S]$ equals

$$= \begin{cases} H(1+q)S(E - S/\Gamma\sigma^2) - I(\theta - S)^2/\Gamma\sigma^2 k & \text{if } S \leqslant \theta, \\ H(1+q)S(E - S/\Gamma\sigma^2) & \text{if } \theta \leqslant S \leqslant e, \\ 0 & \text{if } S \geqslant e. \end{cases}$$

The competitive bid–ask spread S_{NSF} is in the first region and satisfies

$$(1+q)S(e - S) = J(\theta - S)^2. \tag{A.1}$$

(Although bid–ask spreads above e generate zero profits, they do so trivially, generating zero trade.)

With spread futures, the break-even condition for the spread futures market, $HqD \times x[S, D] = 0$ implies $D_{\text{SF}} = 0$. Market-maker profit in the primary futures market $HS \times x[S, 0] + I(S - \theta) \cdot y[S]$ therefore equals

$$= \begin{cases} HS[E - S/\Gamma\sigma^2(1+q)] - I(\theta - S)^2/\Gamma\sigma^2 k & \text{if } S \leqslant \theta, \\ HS[E - S/\Gamma\sigma^2(1+q)] & \text{if } \theta \leqslant S \leqslant (1+q)e, \\ 0 & \text{if } S \geqslant (1+q)e. \end{cases}$$

The competitive bid–ask spread S_{SF} is in the first region and satisfies

$$S \cdot [e - S/(1+q)] = J(\theta - S)^2. \tag{A.2}$$

In comparing (A.1) to (A.2), they share the same quadratic (right-hand side) cost function and range $0 \leqslant S \leqslant \theta$. The quadratic (left-hand side) revenue function of (A.2) is the (left-hand side) revenue function of (A.1) shifted horizontally to the right. To see this, substitute $S' = (1+q)S$ into (A.1) and compare to (A.2). The revenue functions of (A.1) and (A.2) intersect at $S = (1+q)e/(2+q)$. If the shared cost function lies below the intersection of the revenue functions, then the revenue/cost intersection of (A.1) occurs to the left of the revenue/cost intersection of (A.2), which occurs to the left of the revenue/revenue intersection; that is, $S_{\text{NSF}} \leqslant S_{\text{SF}} \leqslant (1+q)e/(2+q)$. If the shared cost function lies above the intersection of the revenue functions, then the revenue/cost intersection of (A.1) occurs to the right of the revenue/cost intersection of (A.2), which occurs to the right of the revenue/revenue intersection; that is, $S_{\text{NSF}} \geqslant S_{\text{SF}} \geqslant (1+q)e/(2+q)$. The relevant condition reduces to whether $J \cdot [\theta - (1+q)e/(2+q)]^2$ is lesser or greater than $[(1+q)e/(2+q)]^2$, which reduces to whether θ is lesser or greater than $[1 + \sqrt{(kH/I)}] \cdot [(1+q)/(2+q)] \cdot \Gamma\sigma^2 E$.

To compare the total hedging volume, $H(1+q)x$, note that with spread futures, $x[S_{\text{SF}}, 0] = E - S_{\text{SF}}/\Gamma\sigma^2(1+q)$, and that without spread futures, $x[S_{\text{NSF}}, S_{\text{NSF}}] = E - S_{\text{NSF}}/\Gamma\sigma^2$. Therefore, compare S_{SF} to $(1+q)S_{\text{NSF}}$. Transform (A.1) by substituting $S' = (1+q)S$ to get

$$S' \cdot [e - S'/(1+q)] = J[\theta - S'/(1+q)]^2 \tag{A.3}$$

over the range $0 \leqslant S' \leqslant (1+q)\theta$.

In comparing (A.2) and (A.3), they share the same revenue functions. The cost function of (A.3) is the cost function of (A.2) stretched horizontally to the right by a factor $(1+q)$. The revenue/cost intersection of (A.3) occurs to the right of the revenue/cost intersection of (A.2), that is, $(1+q)S_{\text{NSF}} \geqslant S_{\text{SF}}$. Total hedging volume is (weakly) higher with spread futures.

To examine the hedger welfare change from introducing spread futures, since $x' = x$ is optimal, substitute into the hedger optimization (1) to get

$$\underset{x}{\text{Max}} \ -(S + qD)|x| - \Gamma\sigma^2(1+q)(x-E)^2/2. \tag{A.4}$$

With spread futures, the total transaction cost $(S + qD) = S_{\text{SF}}$, and without spread futures, $(S + qD) = (1+q)S_{\text{NSF}}$. Since $(1+q)S_{\text{NSF}} \geqslant S_{\text{SF}}$, hedgers are (weakly) better off with spread futures. $\quad\square$

Proof of Proposition 2. The initial futures position of hedgers $x[S, D]$ and informed traders $y[S]$ are given in the proof of Proposition 1. For the case without spread futures, aggregate market-maker profit from (5) is optimized by

$$\underset{S}{\text{Max}} \ (1+q)S(e - S) - J\big(\max(\theta - S, 0)\big)^2. \tag{A.5}$$

The first-order condition implies the optimal spread equals

$$S_{\text{NSF}}^* = \begin{cases} [(1+q)e + 2J\theta]/[2(1+q+J)] & \text{if } e/2 \leqslant \theta \leqslant e, \\ e/2 & \text{if } \theta \leqslant e/2. \end{cases} \tag{A.6}$$

Note that $\theta \leqslant e$ is the assumption previously made to guarantee that markets do not shut down from severe adverse selection.

For the case with spread futures, aggregate market-maker profit from (6) is optimized by

$$\underset{D \leqslant S}{\text{Max}} \ (S + qD)\big[e - (S + qD)/(1 + q)\big] - J\big(\max(\theta - S, 0)\big)^2.$$

For any $D > 0$, the same revenue with lower adverse selection costs can be achieved by raising S and lowering D to keep $(S + qD)$ constant. Therefore, the optimal $D_{\text{SF}}^* = 0$. Substitute this back and simplify to

$$\underset{S}{\text{Max}} \ S\big[e - S/(1 + q)\big] - J\big(\max(\theta - S, 0)\big)^2. \tag{A.7}$$

The first-order condition implies the optimal spread equals

$$S_{\text{SF}}^* = \begin{cases} (1 + q)(e + 2J\theta)/[2(1 + J + qJ)] & \text{if } (1 + q)e/2 \leqslant \theta \leqslant e, \\ (1 + q)e/2 & \text{if } \theta \leqslant (1 + q)e/2. \end{cases} \tag{A.8}$$

To compare S_{NSF}^* and S_{SF}^*, consider the marginal revenue/marginal cost intersections for both (A.5) and (A.7). In this context, marginal is with respect to changes in the price (the bid–ask spread). In the region where marginal revenue is negative (note that marginal cost is), marginal revenue for (A.5), $(1 + q)(e - 2S)$, lies to the left of marginal revenue for (A.7), $e - 2S/(1 + q)$. (A.5) and (A.7) share the same marginal cost function; its intersection with (A.5) marginal revenue is left of its intersection with (A.7) marginal revenue, $S_{\text{NSF}}^* \leqslant S_{\text{SF}}^*$.

To compare the total hedging volume, $H(1 + q)x$, note that with spread futures, $x[S_{\text{SF}}^*, 0] = E - S_{\text{SF}}^*/\Gamma\sigma^2(1 + q)$, and that without spread futures, $x[S_{\text{NSF}}^*, S_{\text{NSF}}^*] = E - S_{\text{NSF}}^*/\Gamma\sigma^2$. Therefore, compare S_{SF}^* to $(1 + q)S_{\text{NSF}}^*$. Transform (A.7) by substituting $S' = S/(1 + q)$,

$$\underset{S'}{\text{Max}} \ (1 + q)S'(e - S') = J \cdot \big(\max(\theta - (1 + q)S', 0)\big)^2. \tag{A.9}$$

The marginal cost of (A.9), $-2J(1 + q) \cdot \max(\theta - (1 + q)S', 0)$, lies above the marginal cost of (A.5), $-2J \cdot \max(\theta - S, 0)$, for values above $\theta/(2 + q)$. Both (A.5) and (A.9) share the same marginal revenue, which is decreasing and non-negative at $\theta/(2 + q)$. Therefore, the marginal revenue/marginal cost intersection of (A.9) is left of the marginal revenue/marginal cost intersection of (A.5), so $S_{\text{SF}}^*/(1 + q) = S_{\text{NSF}}^*$. Total hedging volume is (weakly) higher with spread futures.

For the hedger welfare change from introducing spread futures, refer to (A.4) in Proposition 1. Since $(1 + q)S_{\text{NSF}}^* \geqslant S_{\text{SF}}^*$, hedgers are (weakly) better off with spread futures. □

Proof of Proposition 3. The aggregate market-maker profit π_{NSF}^* in the case of the monopolistic exchange, without spread futures, can be found by substituting (A.6) into (A.5). Thus,

$$\pi_{\text{NSF}}^* = \begin{cases} (H/\Gamma\sigma^2)(-J\theta^2 + [(1 + q)e + 2J\theta]^2/4(1 + q + J)) & \text{if } e/2 \leqslant \theta \leqslant e, \\ (H/\Gamma\sigma^2)(1 + q)e^2/4 & \text{if } \theta \leqslant e/2. \end{cases}$$

The aggregate market-maker profit π_{SF}^* in the case of the monopolistic exchange, with spread futures, can be found by substituting (A.8) into (A.7). Thus,

$$\pi_{\mathrm{SF}}^* = \begin{cases} (H/\Gamma\sigma^2)(-J\theta^2 + (e + 2J\theta)^2/4[J + (1+q)^{-1}] & \text{if } (1+q)e/2 \leqslant \theta \leqslant e, \\ (H/\Gamma\sigma^2)(1+q)e^2/4 & \text{if } \theta \leqslant (1+q)e/2. \end{cases}$$

The difference $\Delta\pi^* = \pi_{\mathrm{SF}}^* - \pi_{\mathrm{NSF}}^*$ gives the increase in market-maker profit from introducing spread futures. Thus, $(\Gamma\sigma^2/H) \cdot \Delta\pi^*$ equals

$$= \begin{cases} (e + 2J\theta)^2/4[J + (1+q)^{-1}] - [(1+q)e + 2J\theta]^2/4(1+q+J) \\ \quad = qJ[4\theta e(1-J) - (2+q)e^2 + 4J\theta^2(2+q)/(1+q)]/ \\ \quad\quad 4(1+q+J)[J + (1+q)^{-1}], & \text{if } (1+q)e/2 \leqslant \theta \leqslant e, \\ (1+q)e^2/4 + J\theta^2 - [(1+q)e + 2J\theta]^2/4(1+q+J) \\ \quad = (1+q)J(2\theta - e)^2/4(1+q+J), & \text{if } e/2 \leqslant \theta \leqslant (1+q)e/2, \\ 0 & \text{if } \theta \leqslant e/2. \end{cases} \quad (\mathrm{A.10})$$

First examine the first region, $\theta \leqslant e \leqslant 2\theta/(1+q)$, where

$$\Delta\pi^* = \left(qHJ/4\Gamma\sigma^2(1+q+J)[J + (1+q)^{-1}]\right)$$
$$\times \left[4\theta e(1-J) - (2+q)e^2 + 4J\theta^2(2+q)/(1+q)\right].$$

Examine the signs of a series of partial derivatives. First with respect to $\Gamma\sigma^2$ (note that Γ and σ^2 only appear together),

$$\partial(\Delta\pi^*)/\partial\left(\Gamma\sigma^2\right)$$
$$= \left(qHJ/4\left(\Gamma\sigma^2\right)^2(1+q+J)[J + (1+q)^{-1}]\right)$$
$$\times \left[-(2+q)e^2 - 4J\theta^2(2+q)/(1+q)\right] \leqslant 0.$$

Therefore, $\Delta\pi^*$ is decreasing in Γ and in σ. Next with respect to E,

$$\partial(\Delta\pi^*)/\partial E$$
$$= \left(qHJ/4(1+q+J)[J + (1+q)^{-1}]\right) \cdot \left[4\theta(1-J) - (2+q)2e\right]$$
$$\leqslant \left(qHJ/4(1+q+J)[J + (1+q)^{-1}]\right) \cdot \left[4e(1-J) - (2+q)2e\right]$$
$$= \left(qHJ/4(1+q+J)[J + (1+q)^{-1}]\right) \cdot \left[-2e(q + 2J)\right] \leqslant 0.$$

$\Delta\pi^*$ is decreasing in E. Next with respect to θ,

$\partial(\Delta\pi^*)/\partial\theta$
$$= \left(qHJ/4\Gamma\sigma^2(1+q+J)\left[J+(1+q)^{-1}\right]\right)$$
$$\times \left[4e(1-J)+8J\theta(2+q)/(1+q)\right]$$
$$\geqslant \left(qHJ/4\Gamma\sigma^2(1+q+J)\left[J+(1+q)^{-1}\right]\right)\cdot\left[4e(1-J)+4eJ(2+q)\right]$$
$$= \left(qHJ/4\Gamma\sigma^2(1+q+J)\left[J+(1+q)^{-1}\right]\right)\cdot\left[4e(1+J+qJ)\right]\geqslant 0.$$

$\Delta\pi^*$ is increasing in θ. Next with respect to J,

$$\partial(\Delta\pi^*)/\partial J = \left(qH/4\Gamma\sigma^2(1+q+J)^2\left[J+(1+q)^{-1}\right]^2\right)$$
$$\times \left(\left[(2+q)(2\theta-e)^2+4\theta^2(2+q)/(1+q)^2-4\theta e/(1+q)\right]J^2\right.$$
$$\left.+\left[8\theta^2(2+q)/(1+q)-8\theta e\right]J+\left[4\theta e-(2+q)e^2\right]\right)$$
$$\geqslant \left(qH/4\Gamma\sigma^2(1+q+J)^2\left[J+(1+q)^{-1}\right]^2\right)$$
$$\times \left(\left[(2+q)(2\theta-e)^2+2\theta e(2+q)/(1+q)-4\theta e/(1+q)\right]J^2\right.$$
$$\left.+\left[4\theta e(2+q)-8\theta e\right]J+\left[2(1+q)e^2-(2+q)e^2\right]\right)$$
$$= \left(qH/4\Gamma\sigma^2(1+q+J)^2\left[J+(1+q)^{-1}\right]^2\right)$$
$$\times \left(\left[(2+q)(2\theta-e)^2+2\theta eq/(1+q)\right]J^2+4\theta eqJ+qe^2\right)\geqslant 0.$$

Since $J=(I/kH)$, $\Delta\pi^*$ is increasing in I and decreasing in k. Next, write $\Delta\pi^*$ as

$$\Delta\pi^* = \left(qI/4k\Gamma\sigma^2(1+q+J)\left[J+(1+q)^{-1}\right]\right)$$
$$\times \left[4\theta e(1-J)-(2+q)e^2+4J\theta^2(2+q)/(1+q)\right]$$

and take the partial derivative with respect to H,

$\partial(\Delta\pi^*)/\partial H$
$$= \left(qI/4k\Gamma\sigma^2(1+q+J)^2\left[J+(1+q)^{-1}\right]^2\right)\cdot(\partial J/\partial H)$$
$$\times \left(\left[J^2+(2+2q+q^2)J/(1+q)+1\right]\left[4\theta^2(2+q)/(1+q)-4\theta e\right]\right.$$
$$-\left[4\theta e(1-J)-(2+q)e^2+4J\theta^2(2+q)/(1+q)\right]$$
$$\left.\times\left[2J+(2+2q+q^2)/(1+q)\right]\right)$$
$$= \left(qI^2/4kH\Gamma\sigma^2(1+q+J)^2\left[J+(1+q)^{-1}\right]^2\right)$$
$$\times \left(\left[4\theta^2(2+q)/(1+q)-4\theta e\right]J^2+2\left[4\theta e-(2+q)e^2\right]J\right.$$
$$\left.+\left[-(2+q)(2+2q+q^2)e^2+4\theta(3+3q+q^2)e-4\theta^2(2+q)\right]\cdot(1+q)^{-1}\right).$$

The coefficients of J^2, J, and $(1+q)^{-1}$ are all positive. To show this for J^2,

$$4\theta^2(2+q)/(1+q)-4\theta e\geqslant 2\theta(2+q)e-4\theta e=2\theta qe\geqslant 0.$$

To show this for J,

$$2\left[4\theta e-(2+q)e^2\right]\geqslant 2\left[2(1+q)e^2-(2+q)e^2\right]=2qe^2\geqslant 0.$$

To show this for $(1+q)^{-1}$ across the region $\theta \leqslant e \leqslant 2\theta/(1+q)$, note that this term is quadratic in e, with a negative coefficient on e^2. Therefore, the minimum value over all possible values of e occurs at an endpoint, either at $e = \theta$ or $e = 2\theta/(1+q)$. At $e = \theta$, the term has value $(2-q^2)q\theta^2 \geqslant 0$, and at $e = 2\theta/(1+q)$, the term has value $4\theta^2 q/(1+q)^2 \geqslant 0$. Therefore, the term is non-negative over the region, and $\Delta\pi^*$ is increasing in H.

Now examine the second region, $2\theta/(1+q) \leqslant e \leqslant 2\theta$, where

$$\Delta\pi^* = (1+q)HJ(2\theta - e)^2/4\Gamma\sigma^2(1+q+J).$$

It follows that $\Delta\pi^*$ is decreasing in E, increasing in θ, increasing in J (and therefore increasing in I and decreasing in k), and increasing in q. Taking the partial derivative with respect to $\Gamma\sigma^2$,

$$\partial(\Delta\pi^*)/\partial\left(\Gamma\sigma^2\right)$$
$$= \left[(1+q)HJ/4\left(\Gamma\sigma^2\right)^2(1+q+J)\right] \cdot \left[-2e(2\theta - e) - (2\theta - e)^2\right] \leqslant 0.$$

Therefore, $\Delta\pi^*$ is decreasing in Γ and in σ. Taking the partial derivative with respect to H,

$$\partial(\Delta\pi^*)/\partial H = \left[(1+q)(I/k)(2\theta - e)^2/4\Gamma\sigma^2(1+q+J)^2\right] \cdot \left[I/kH^2\right] \geqslant 0.$$

$\Delta\pi^*$ is increasing in H.   □

Proof of Proposition 4. Note that $\Delta P_{t+1} = \rho\Delta P_t + \varepsilon_{t+1}$, so at time t, ε_{t+1} is the unexpected part of ΔP_{t+1}. Also, $(\Delta P_{t+1} + \Delta P_{t+2}) = \rho(1+\rho)\Delta P_t + (1+\rho)\varepsilon_{t+1} + \varepsilon_{t+2}$, so at time t, $(1+\rho)\varepsilon_{t+1} + \varepsilon_{t+2}$ is the unexpected part of $(\Delta P_{t+1} + \Delta P_{t+2})$. Equation (7) follows.

Optimizing (7), and assuming nonzero demand, the hedger futures position magnitudes are

$$x'[S, D, x] = \begin{cases} (e-D)/\Gamma\sigma^2 & \text{if } x \geqslant (e-D)/\Gamma\sigma^2, \\ (e-S)/\Gamma\sigma^2 & \text{if } x \leqslant (e-S)/\Gamma\sigma^2, \\ x & \text{otherwise;} \end{cases}$$

$$x[S, D] = \begin{cases} ((1+\rho q)e - S)/\Gamma\sigma^2 & \text{if } S - D \leqslant \rho qe, \\ ((1+\rho q)e - (1-q)S - qD)/\Gamma\sigma^2 & \text{if } S - D \leqslant -\rho e, \\ ((1+q+\rho q)e - S - qD)/(1+q)\Gamma\sigma^2 & \text{otherwise.} \end{cases}$$

The first and third cases are possible when $\rho > 0$, and the second and third when $\rho < 0$.

In the competitive case without spread futures, $D = S$. For all ρ, $x = ((1+\rho q)e - S)/\Gamma\sigma^2$ and $x' = (e - S)/\Gamma\sigma^2$. Aggregate market-maker profit $H(Sx + qDx') + I(S - \theta) \cdot y$ equals zero when the bid–ask spread S_{NSF} satisfies

$$S \cdot \left((1+q+\rho q)e - (1+q)S\right) = J(\theta - S)^2. \tag{A.11}$$

Hedging volume $H(x + qx') = (H/\Gamma\sigma^2) \cdot ((1+q+\rho q)e - (1+q)S_{\text{NSF}})$.

In the competitive case with spread futures, break-even for the spread futures market implies $D_{\text{SF}} = 0$. The hedge positions satisfy

$$
\begin{aligned}
&x = \big((1+\rho q)e - S\big)/\Gamma\sigma^2, \quad x' = e/\Gamma\sigma^2 && \text{if } \rho \geqslant 0 \text{ and } S \leqslant \rho q e, \\
&x = x' = \big((1+q+\rho q)e - S\big)/(1+q)\Gamma\sigma^2 && \text{if } \rho \geqslant 0 \text{ and } S \geqslant \rho q e, \\
&x = \big((1+\rho q)e - (1-q)S\big)/\Gamma\sigma^2, \quad x' = (e-S)/\Gamma\sigma^2 && \text{if } \rho \leqslant 0 \text{ and } S \leqslant -\rho e, \\
&x = x' = \big((1+q+\rho q)e - S\big)/(1+q)\Gamma\sigma^2 && \text{if } \rho \leqslant 0 \text{ and } S \geqslant -\rho e.
\end{aligned}
$$

For $\rho \geqslant 0$, $x \geqslant x'$, and aggregate market-maker profit $H S x + I(S-\theta)y$ equals zero when the bid–ask spread S_{SF} satisfies

$$
S \cdot \Gamma\sigma^2 x = J(\theta - S)^2. \tag{A.12}
$$

For $\rho < 0$, $x' \geqslant x$, and aggregate market-maker profit $H S(x + q(x' - x)) + I(S-\theta)y$ equals zero when the bid–ask spread S_{SF} satisfies

$$
S \cdot \Gamma\sigma^2 \big((1-q)x + q x'\big) = J(\theta - S)^2. \tag{A.13}
$$

In both cases, total hedging volume is $H(x + q x') = (H/\Gamma\sigma^2)((1+q+\rho q)e - S_{\mathrm{SF}})$.

For $\rho \geqslant 0$, compare (A.11) and (A.12). The cost functions are identical. The revenue functions intersect at $S = (1+q+\rho q)e/(2+q)$. Whether the common cost function lies above or below the intersection of revenue functions determines whether the revenue/cost intersection of (A.11) occurs to the left or right of the revenue/cost intersection of (A.12). This condition reduces to $S_{\mathrm{SF}} > S_{\mathrm{NSF}}$ when $\theta < [1 + \sqrt{(1/J)}] \cdot (1+q+\rho q)e/(2+q)$.

Similarly, for $\rho < 0$, compare (A.11) and (A.13). The cost functions are identical. The revenue functions intersect at $S = (1+q+\rho q)e/(2+q)$ for the $\rho \geqslant -1/2$ case, and $S = (1+\rho q)e/(2-q)$ for the $\rho < -1/2$ case. Checking whether the common cost function lies above or below the intersection of revenue functions yields parametric conditions reducing to $S_{\mathrm{SF}} > S_{\mathrm{NSF}}$ when $\theta < [1 + \sqrt{(1/J)}] \cdot (1+q+\rho q)e/(2+q)$ for $\rho \geqslant -1/2$, and $\theta < [1 + \sqrt{(1-(1+2\rho)q^2/(1+\rho q)J)}] \cdot (1+\rho q)e/(2-q)$ for $\rho < -1/2$.

To compare total hedging volumes with and without spread futures, one need only compare S_{SF} to $(1+q)S_{\mathrm{NSF}}$. Transform (A.11) by substituting $M = (1+q)S$ to get

$$
M\big((1+q+\rho q)e - M\big)/(1+q) = J\big[\theta - M/(1+q)\big]^2. \tag{A.14}
$$

For $\rho \geqslant 0$, compare (A.12) and (A.14). The cost function of (A.14) is the cost function of (A.12) stretched horizontally to the right by a factor $(1+q)$; they are non-negative and declining. The revenue function of (A.12) is above the revenue function of (A.14), and starts at the origin. This implies that the revenue/cost intersection of (A.12) occurs to the left of the revenue/cost intersection of (A.14), that is, $S_{\mathrm{SF}} \leqslant (1+q)S_{\mathrm{NSF}}$. A similar argument holds for $\rho < 0$, comparing (A.13) and (A.14). Total hedging volume is (weakly) higher with spread futures.

By numerical calculation over possible parameters, hedger welfare is (weakly) higher with spread futures. $\square$

Proof of Proposition 5. For the case without spread futures, optimization (10) can be rewritten as:

$$
\operatorname*{Max}_{S} \ S\big((1+q+\rho q)e - S\big) - J\big(\max(\theta - S, 0)\big)^2, \tag{A.15}
$$

with total hedging volume $H(x + qx') = (H/\Gamma\sigma^2)((1 + q + \rho q)e - S^*_{NSF})$. This has marginal revenue $(1 + q + \rho q)e - 2S$, and marginal cost $-2J \cdot \max(\theta - S, 0)$.

For the case with spread futures, optimization (11) can be rewritten as:

$$\underset{D \leqslant S}{\text{Max}} \, (1 - q)Sx + qS \cdot \max(x, x') + qD \cdot \min(x, x') - J \cdot \big(\max(\theta - S, 0)\big)^2. \quad \text{(A.16)}$$

For $\rho \geqslant 0$, the revenue terms can be written as:

$$= \begin{cases} (1 + \rho q)eS - S^2 + qD(e - D) & \text{if } S - D \leqslant \rho qe, \\ ((1 + q + \rho q)e(S + qD) - (S + qD)^2)/(1 + q) & \text{if } S - D \geqslant \rho qe. \end{cases}$$

For $\rho \leqslant 0$, the revenue terms can be written as:

$$= \begin{cases} (1 + \rho q)e((1 - q)S + qD) - ((1 - q)S + qD)^2 \\ \quad + qS(e - S), & \text{if } S - D \leqslant -\rho e, \\ ((1 + q + \rho q)e(S + qD) - (S + qD)^2)/(1 + q) & \text{if } S - D \geqslant -\rho e. \end{cases}$$

For $\theta < (1 + q + \rho q)e/2(1 + q)$, the optimal bid–ask spread without spread futures is $S^*_{NSF} = (1 + q + \rho q)e/2(1 + q)$, with hedger welfare $-\rho^2 q(1 - q)e^2/2 - 3(1 + q + \rho q)^2 e^2/8(1 + q)$. The optimal bid–ask spreads with spread futures are $D^*_{SF} = e/2$ and $S^*_{SF} = (1 + \rho q)e/2$, with hedger welfare $-\rho^2 q(1 - q)e^2/2 - 3(q + (1 + \rho q)^2)e^2/8$. In this case, hedgers are better off without spread futures. In other cases, for example when $\rho = 0$, hedgers are better off with spread futures. Therefore, D^*_{SF} need not be zero and hedger welfare can go either way.

By numerical calculation over possible parameters, $S^*_{SF} \geqslant S^*_{NSF}$, and $(1 + q)S^*_{NSF} \geqslant S^*_{SF} + qD^*_{SF}$, so total hedging volume is (weakly) higher with spread futures. □

Proof of Proposition 6. Optimizing (12) over x', and writing $A = 4\gamma Q/(\Gamma\sigma^2)^2$, the adjusted hedger futures position magnitudes are:

$$x' = \begin{cases} E - (2/A\Gamma\sigma^2)(-1 + \sqrt{(1 + AD)}), \\ \quad \text{if } x > E - (2/A\Gamma\sigma^2)(-1 + \sqrt{(1 + AD)}), \\ E - (2/A\Gamma\sigma^2)(-1 + \sqrt{(1 + AS)}), \\ \quad \text{if } x < E - (2/A\Gamma\sigma^2)(-1 + \sqrt{(1 + AS)}), \\ x \quad \text{otherwise.} \end{cases}$$

Optimizing over x, only the third case of x' turns out to be relevant, and $x = x' = x_1$, where $x_1 = E - (2/A\Gamma\sigma^2)(-1 + \sqrt{(1 + AZ)})$, and $Z = (S + qD)/(1 + q)$. A similar calculation applies for other endowment and coefficient values, with $x_2 = E - (2/A\Gamma\sigma^2)(1 - \sqrt{(1 - AZ)})$. To guarantee x_1 and x_2 to be non-negative, it is required that $-1 < AZ < 1$, and that $Z < e - Ae^2/4$. Aggregating over the four hedger types (endowments $\pm E$, coefficients $\pm\gamma$) gives

$$x[S, D] = \big(e - f\big((S + qD)/(1 + q)\big)\big)/\Gamma\sigma^2,$$

where $f(Z) = (\sqrt{(1 + AZ)} - \sqrt{(1 - AZ)})/A$. Note that f is increasing. In the competitive case without spread futures, $D = S$. Aggregate market-maker profit $H(Sx + qDx') +$

$I(S - \theta)y$ equals zero when the bid–ask spread S_{NSF} satisfies

$$(1 + q)S\bigl(e - f(S)\bigr) = J(\theta - S)^2. \tag{A.17}$$

Total hedging volume $H(x + qx') = (H/\Gamma\sigma^2)(1 + q)(e - f(S_{\text{NSF}}))$.

In the competitive case with spread futures, break-even for the spread futures market implies $D_{\text{SF}} = 0$. Aggregate market-maker profit in the primary market $HSx + I(S - \theta)y$ equals zero when the bid–ask spread S_{SF} satisfies

$$S\bigl(e - f\bigl(S/(1 + q)\bigr)\bigr) = J(\theta - S)^2. \tag{A.18}$$

Total hedging volume $H(x + qx') = (H/\Gamma\sigma^2)(1 + q)(e - f(_{\text{SF}}/(1 + q)))$.

Revenue function (A.18) is revenue function (A.17) stretched rightward by a factor $(1 + q)$. (Both are upside-down U-shaped.) Consider their (non-origin) intersection point. If the shared cost function of (A.17) and (A.18) passes below/above the revenue functions at the intersection, then S_{NSF} is less/greater than S_{SF}. A sufficient condition to pass below are that the maximum value of the cost function, $J\theta^2$, is less than the revenue value at the intersection, i.e., J or θ is sufficiently small. Sufficient conditions to pass above are that θ lies to the right of the intersection (θ sufficiently large) and that the cost function is very steep (J sufficiently large).

Hedging volumes with and without spread futures are compared by comparing S_{SF} with $(1 + q)S_{\text{NSF}}$. Transform (A.17) by substituting $M = (1 + q)S$ to get

$$M\bigl(e - f\bigl(M/(1 + q)\bigr)\bigr) = J\bigl(\theta - M/(1 + q)\bigr)^2. \tag{A.19}$$

Compare (A.18) and (A.19). They share revenue functions. The cost function of (A.19) is above that of (A.18), implying the revenue/cost intersection of (A.19) occurs to the right of that for (A.18), implying $(1 + q)S_{\text{NSF}} \geqslant S_{\text{SF}}$. Total hedging volume is (weakly) higher with spread futures. Substituting $x' = x$ into (12) shows that hedger welfare depends upon $(S + qD)$; this is S_{SF} with spread futures, and $(1 + q)S_{\text{NSF}}$ without. Hedger welfare is therefore (weakly) higher with spread futures. $\quad\square$

Proof of Proposition 7. For the case without spread futures, market-maker profit is optimized by maximizing

$$(1 + q)S\bigl(e - f(S)\bigr) - J\bigl(\max(\theta - S, 0)\bigr)^2, \tag{A.20}$$

with total hedging volume $H(x + qx') = (H/\Gamma\sigma^2)(1 + q)(e - f(S_{\text{NSF}}^*))$. This leads to marginal revenue $(1 + q)(e - f(S) - Sf'(S))$ and marginal cost $-2J \cdot \max(\theta - S, 0)$.

For the case with spread futures, market-maker profit is optimized by maximizing

$$(S + qD)\bigl(e - f\bigl((S + qD)/(1 + q)\bigr)\bigr) - J\bigl(\max(\theta - S, 0)\bigr)^2. \tag{A.21}$$

For a given level of $(S + qD)$, this can be (weakly) increased by increasing S and decreasing D, while keeping $(S + qD)$ fixed. It follows that $D_{\text{SF}}^* = 0$, and (A.21) can be rewritten as:

$$S\bigl(e - f\bigl(S/(1 + q)\bigr)\bigr) - J\bigl(\max(\theta - S, 0)\bigr)^2, \tag{A.22}$$

with total hedging volume $H(x + qx') = (H/\Gamma\sigma^2)(1 + q)(e - f(S_{SF}^*/(1 + q)))$. This leads to marginal revenue $(e - f(S/(1 + q)) - (S/(1 + q))f'(S/(1 + q)))$ and marginal cost $-2J \cdot \max(\theta - S, 0)$.

Note that, because $-2f'(S) - Sf''(S) < 0$, marginal revenue is decreasing. Relative to the function $(e - f(S) - Sf'(S))$, marginal revenue in (A.20) is stretched vertically by a factor $(1 + q)$, and marginal revenue in (A.22) is stretched to the right by a factor $(1 + q)$. In the relevant region (non-positive marginal revenue), marginal revenue for (A.20) is left of marginal revenue for (A.22). The shared marginal cost function (which starts out below the marginal revenues) intersects the marginal revenue of (A.20) left of where it intersects the marginal revenue of (A.22), so $S_{NSF}^* \leqslant S_{SF}^*$.

To compare hedging volumes, let $M = (1 + q)S$ in (A.20); rewrite as

$$M\big(e - f\big(M/(1 + q)\big)\big) - J\big(\max\big(\theta - M/(1 + q), 0\big)\big)^2. \tag{A.23}$$

Marginal revenue is $(e - f(M/(1 + q)) - (M/(1 + q))f'(M/(1 + q)))$, and marginal cost is $-2J \cdot \max(\theta - M/(1 + q), 0)/(1 + q)$. The marginal revenue functions in (A.22) and (A.23) are identical and downward sloping. The marginal cost functions of (A.22) and (A.23) intersect at $\theta/(2 + q)$; to the right, the (A.22) marginal cost function is greater. If the marginal revenue function is greater than the marginal cost functions at $\theta/(2 + q)$, then the marginal revenue/cost intersection for (A.22) is to the left of the intersection for (A.23); that is, $S_{SF}^* \leqslant (1 + q)S_{NSF}^*$.

Write $\phi = A\theta/(2 + q)$. Note $\phi \leqslant A\theta/2 \leqslant A(e - Ae^2/4)/2$. Inverting, $Ae \geqslant 2 - 2\sqrt{(1 - 2\phi)}$. At $\theta/(2 + q)$, marginal revenue less cost equals

$$
\begin{aligned}
& e - \big(\sqrt{(1 + \phi)} - \sqrt{(1 - \phi)} + \phi/2\sqrt{(1 + \phi)} + \phi/2\sqrt{(1 - \phi)}\big)/A \\
& \quad + 2J\theta(1 + q)/(2 + q) \\
& \geqslant \big(2 - 2\sqrt{(1 - 2\phi)} - (\sqrt{(1 + \phi)} - \sqrt{(1 - \phi)} + \phi/2\sqrt{(1 + \phi)} \\
& \quad + \phi/2\sqrt{(1 - \phi)} + 0)/A,
\end{aligned}
$$

which is non-negative (it equals 0 at $\phi = 0$; and is increasing in ϕ). It follows that $S_{SF}^* \leqslant (1 + q)S_{NSF}^*$, so total hedging volume is (weakly) higher with spread futures. Also, since $x' = x$, hedger welfare is (weakly) higher with spread futures. $\quad\square$

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